**Measures of Center and Spread**

**Common Core Math Standards**

The student is expected to:

S-ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

**Mathematical Practices**

MP.1 Problem Solving

**Language Objective**

Explain the difference between a measure of center and a measure of spread.

**ENGAGE**

**Essential Question:** How can you describe and compare data sets?

Use the mean and median to describe and compare the center of data sets. Use the range, interquartile range, or standard deviation to describe and compare the spread of data sets.

**PREVIEW: LESSON PERFORMANCE TASK**

View the Engage section online. Discuss who might participate in a town choir, and describe the four sections that typically make up a choir: soprano, alto, tenor, and bass. Then preview the Lesson Performance Task.

**EXPLORE Exploring Data**

Caleb and Kim have bowled three games. Their scores are shown in the chart below.

<table>
<thead>
<tr>
<th>Name</th>
<th>Game 1</th>
<th>Game 2</th>
<th>Game 3</th>
<th>Average Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Caleb</td>
<td>151</td>
<td>153</td>
<td>146</td>
<td>150</td>
</tr>
<tr>
<td>Kim</td>
<td>122</td>
<td>139</td>
<td>189</td>
<td>150</td>
</tr>
</tbody>
</table>

Complete the table.

1. Find Caleb's average score over the three games and enter it in the table.
   
   \[
   \frac{151 + 153 + 146}{3} = \frac{450}{3} = 150
   \]

2. Find Kim's average score over the three games and enter it in the table.
   
   \[
   \frac{122 + 139 + 189}{3} = \frac{450}{3} = 150
   \]

3. How do their average scores compare?
   - Caleb's scores are farther from / closer to the average than Kim's.

4. They bowl a fourth game, where Caleb scores 150 and Kim scores a 175. How does this affect their averages?
   - Caleb's average stays the same. Kim's average increases.

5. Does the Game 4 score affect the consistency of their scores? Explain.
   - No, Caleb's scores are still consistent, and Kim's scores are still inconsistent.
Reflect

1. **Discussion** Is the average an accurate representation of Caleb’s bowling?
   
   **Yes, Caleb is a consistent bowler, and his scores are all very close to his average.**

2. **Discussion** Is the average an accurate representation of Kim’s bowling?
   
   **No. Kim is an inconsistent bowler and has a fairly wide range of scores.**

**Explain 1**  Measures of Center: Mean and Median

Two commonly used measures of center for a set of numerical data are the mean and median. Measures of center represent a central or typical value of a data set. The **mean** is the sum of the values in the set divided by the number of values in the set. The **median** is the middle value in a set when the values are arranged in numerical order.

**Example 1** Find the mean and median of each data set.

**A** The number of text messages that Isaac received each day for a week is shown.

47, 49, 54, 50, 48, 47, 55

Find the mean. Divide the sum by the numbers of data values.

$$\frac{350}{7} = 50.$$ The mean is 50 text messages a day.

Find the median. Rewrite the values in increasing order.

47, 47, 48, 49, 50, 54, 55. The median is 49 text messages a day.

**B** The amount of money Elise earned in tips per day for 6 days is listed below.

$75, $97, $360, $84, $119, $100

Find the mean to the nearest $0.01. Divide the sum by the number of data values.

$$\frac{835}{6} = \$139.17.$$ The mean is **$139.17**.

Find the median. Rewrite the values in increasing order.

75, 84, 97, 100, 119, 360

Find the mean of the middle two values.

$$\frac{97 + 100}{2} = 98.5$$

The median is **$98.50**.

**EXPLORE**

Exploring Data

**EXPLAIN 1**  Measures of Center: Mean and Median

**INTEGRATE MATHEMATICAL PRACTICES**

**Focus on Modeling**

**MP.4** Graphing a data set and its mean on a number line can help students visualize how close the values are to the mean value.

**QUESTIONING STRATEGIES**

- **How can you determine the mean value of a data set?** Find the sum of all the numbers, then divide the sum by the number of values in the data set.

**EXPLAIN 1**  Measures of Center: Mean and Median

**INTEGRATE MATHEMATICAL PRACTICES**

**Focus on Critical Thinking**

**MP.3** Ask students what must be true about a data set in order for the median to be one of the values in the data set. Students should understand that when the median is in the data set, either the set has an odd number of values, or the set has an even number of values and the two middle values are the same.

**PROFESSIONAL DEVELOPMENT**

**Integrate Mathematical Practices**

This lesson provides an opportunity to address Mathematical Practices **MP.1**, which calls for students to “make sense of problems and persevere in solving them.” Students learn about statistical measures that describe the center and spread of a set of numerical data. They will perform multi-step calculations to find the mean, median, range, interquartile range, and standard deviation of a data set, and they will use those measures to make sense of data.
QUESTIONING STRATEGIES

What is a situation in which the mean would not be the best measure of center?

Explain. Possible answer: A set of test scores in which the scores are fairly consistent but there is one very low score; the very low score will lower the mean to below the typical value for the data set.

If you had test scores that were mostly clustered around a particular value, but one score was a lot higher, would you prefer that your teacher report your class grade using the mean or the mean of the scores? Explain. I would prefer the mean, because the higher score would raise the mean, but it would not affect the median much.

EXPLAIN 2

Measures of Spread: Range and IQR

AVOID COMMON ERRORS

Students may include the median in the upper and lower halves of the data set when finding Q1 and Q3. Remind them that the median is never included in these halves.

QUESTIONING STRATEGIES

Why must the median be found before the first and third quartiles? The first and third quartiles are the halfway points of the lower half and upper half of the data, respectively. In order to find these, you must first find the halfway point of the data (the median).

COLLABORATIVE LEARNING

Peer-to-Peer Activity

Have students work in pairs. Have each pair write 10 different numbers on separate slips of paper, then evenly divide the slips so that each student has 5 numbers to create a data set. Have students determine the standard deviations for their data sets and compare results with their partners. Ask students how they would select a data set so that the standard deviation is as small as possible. Have them discuss strategies.
The numbers of runs scored by a softball team in 20 games are given.
3, 4, 8, 12, 7, 5, 4, 12, 3, 9, 11, 4, 14, 8, 2, 10, 3, 10, 9, 7

Order the data values.
2, 3, 3, 4, 4, 4, 5, 7, 8, 8, 9, 9, 10, 10, 11, 12, 12, 14

Median = \(\frac{7 + 8}{2} = 7.5\)

Range = 14 - 2 = 12

Interquartile range.
\(Q_1 = \frac{4 + 4}{2} = 4\) and \(Q_3 = \frac{10 + 10}{2} = 10\)

IQR = \(Q_3 - Q_1 = 10 - 4 = 6\)

The median is 7.5.
The range is 12.
The IQR is 6.

Reflect

6. Discussion Why is the IQR less than the range?
The IQR is less than the range because the range is the difference of the two extreme values, the largest and smallest data values. The IQR is the difference between numbers within the ordered set, so unless all of the values in the set are the same, \(Q_1\) will be larger than the smallest element of the set, and \(Q_3\) will be smaller than the largest element of the set.

Your Turn

Find the median, range, and interquartile range for the given data set.

7. 21, 31, 26, 24, 28, 26

21, 24, 26, 26, 28, 31 (median is underlined and quartiles are double underlined)

The median is 26.
Range = 31 - 21 = 10
IQR = 28 - 24 = 4

8. The high temperatures in degrees Fahrenheit on 11 days were 68, 71, 75, 74, 75, 71, 73, 71, 72, 74, and 79.

68, 71, 71, 72, 73, 74, 74, 75, 75, 79

The median is 73.
Range = 79 - 68 = 11
IQR = 75 - 71 = 4

DIFFERENTIATE INSTRUCTION

Critical Thinking

Discuss whether the standard deviation of a data set (the square root of the average squared deviation from the mean) is a better measure of spread than the average unsquared deviation from the mean. Have students calculate and compare the two measures for a few data sets. Students should realize that if deviations are not squared, negative and positive deviations can cancel each other out, so a data set with a large spread could have a very small average deviation. Squaring the deviations makes all the numbers positive, so this does not happen. In addition, squaring gives large deviations more weight than small deviations.
**EXPLAIN 3**

**Measures of Spread: Standard Deviation**

**INTEGRATE MATHEMATICAL PRACTICES**

**Focus on Math Connections**

**MP.1** Discuss the meaning of the standard deviation for a data set. Students should be aware that a low standard deviation means that most data points will be very close to the mean, while a high standard deviation means that the data points are spread out across a large range of values.

**QUESTIONING STRATEGIES**

How do the three measures of spread—range, interquartile range, and standard deviation—differ? How is each one related to measures of center for a data set? The range measures the spread of the entire data set, but it is not based on any measure of center. The IQR depends on the median; it measures the spread of the middle half of the data. The standard deviation measures the spread of the data relative to the mean.

**Explain 3**

**Measures of Spread: Standard Deviation**

Another measure of spread is the standard deviation, which represents the average of the distance between individual data values and the mean.

The formula for finding the standard deviation of the data set \( \{x_1, x_2, x_3, \ldots, x_n\} \), with \( n \) elements and mean \( \overline{x} \), is shown below.

\[
\text{standard deviation} = \sqrt{\frac{(x_1 - \overline{x})^2 + (x_2 - \overline{x})^2 + \ldots + (x_n - \overline{x})^2}{n}}
\]

**A** Find the standard deviation of 77, 86, 84, 93, 90.

Find the mean.

\[
\text{mean} = \frac{77 + 86 + 84 + 93 + 90}{5} = \frac{430}{5} = 86
\]

Complete the table.

<table>
<thead>
<tr>
<th>Data Value, ( x )</th>
<th>Deviation from Mean, ( x - \overline{x} )</th>
<th>Squared Deviation, ( (x - \overline{x})^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>77</td>
<td>77 - 86 = -9</td>
<td>((-9)^2 = 81)</td>
</tr>
<tr>
<td>86</td>
<td>86 - 86 = 0</td>
<td>(0^2 = 0)</td>
</tr>
<tr>
<td>84</td>
<td>84 - 86 = -2</td>
<td>((-2)^2 = 4)</td>
</tr>
<tr>
<td>93</td>
<td>93 - 86 = 7</td>
<td>(7^2 = 49)</td>
</tr>
<tr>
<td>90</td>
<td>90 - 86 = 4</td>
<td>(4^2 = 16)</td>
</tr>
</tbody>
</table>

Find the mean of the squared deviations.

\[
\text{mean squared deviation} = \frac{81 + 0 + 4 + 49 + 16}{5} = \frac{150}{5} = 30
\]

Find the square root of the mean of the squared deviations, rounding to the nearest tenth. \( \sqrt{30} = 5.5 \)

The standard deviation is approximately 5.5.

**B** Find the standard deviation of 3, 4, 8, 12, 7, 5, 4, 12, 3, 9, 11, 4, 14, 8, 2, 10, 3, 10, 9, 7.

Find the mean.

\[
\text{mean} = \frac{2 + 3 + 3 + 4 + 4 + 4 + 5 + 7 + 7 + 8 + 8 + 9 + 9 + 10 + 10 + 11 + 12 + 12 + 14}{20} = \frac{145}{20} = 7.25
\]

**LANGUAGE SUPPORT**

**Connect Vocabulary**

Review the explanations of measures of center, mean, and median. Students may be familiar with the concept of average, but not with the academic term mean, or they may understand that mean is another word for average, but may not grasp the concept of median. Remind them to ask for additional clarification before moving forward if they don't understand the terms. Remind them that they can also use the glossary as a resource.
Complete the table.

<table>
<thead>
<tr>
<th>Data Value, $x$</th>
<th>Deviation from Mean, $x - \bar{x}$</th>
<th>Squared Deviation, $(x - \bar{x})^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$7.25$</td>
<td>$-5.25$</td>
</tr>
<tr>
<td>3</td>
<td>$7.25$</td>
<td>$-4.25$</td>
</tr>
<tr>
<td>3</td>
<td>-4.25</td>
<td>18.0625</td>
</tr>
<tr>
<td>3</td>
<td>-4.25</td>
<td>18.0625</td>
</tr>
<tr>
<td>4</td>
<td>$7.25$</td>
<td>$-3.25$</td>
</tr>
<tr>
<td>4</td>
<td>-3.25</td>
<td>10.5625</td>
</tr>
<tr>
<td>4</td>
<td>-3.25</td>
<td>10.5625</td>
</tr>
<tr>
<td>5</td>
<td>$7.25$</td>
<td>$-2.25$</td>
</tr>
<tr>
<td>7</td>
<td>$7.25$</td>
<td>$-0.25$</td>
</tr>
<tr>
<td>7</td>
<td>-0.25</td>
<td>0.0625</td>
</tr>
<tr>
<td>8</td>
<td>$7.25$</td>
<td>0.75</td>
</tr>
<tr>
<td>8</td>
<td>0.75</td>
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<tr>
<td>9</td>
<td>$7.25$</td>
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<tr>
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<td>1.75</td>
<td>3.0625</td>
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<td>2.75</td>
</tr>
<tr>
<td>10</td>
<td>2.75</td>
<td>7.5625</td>
</tr>
<tr>
<td>11</td>
<td>$7.25$</td>
<td>3.75</td>
</tr>
<tr>
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<td>4.75</td>
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<tr>
<td>12</td>
<td>4.75</td>
<td>22.5625</td>
</tr>
<tr>
<td>14</td>
<td>$7.25$</td>
<td>6.75</td>
</tr>
<tr>
<td>14</td>
<td>6.75</td>
<td>45.5625</td>
</tr>
</tbody>
</table>
ELABORATE

INTEGRATE MATHEMATICAL PRACTICES

Focus on Technology

MP.5  Explain that measures of center and spread can also be found using a computer spreadsheet. If spreadsheet software is available, demonstrate how to enter the data into cells of a spreadsheet and use the statistical functions available within the software.

You may wish to point out that the standard deviation formula taught in this lesson should be used when a data set includes an entire population. A slightly different formula, with \( n - 1 \) in the denominator instead of \( n \), is used when the data set is only a sample of a population. The two formulas are available as different functions in spreadsheet software.

**SUMMARIZE THE LESSON**

How can you determine the first and third quartile of a data set? **First, determine the median.** This allows you to determine the lower and upper half of the data set. The first quartile is the median of the lower half of the data set, and the third quartile is the median of the upper half of the data set. Find the median of each half of the data set the same way you find the median of the entire data set.

Find the mean of the squared deviations.

\[
\text{mean squared deviation} = \frac{27.5625 + 3(18.0625) + 3(10.5625) + 5.0625 + 2(0.0625) + 2(0.0625) + 2(3.0625) + 2(7.5625) + 14.0625 + 2(22.5625) + 45.5625}{20} = \frac{245.75}{20} = 12.2875
\]

Find the square root of the mean of the squared deviations, rounding to the nearest tenth.

\[
\sqrt{12.2875} \approx 3.5
\]

The standard deviation is approximately **3.5**.

<table>
<thead>
<tr>
<th>Data Value, ( x )</th>
<th>Deviation from Mean, ( x - \bar{x} )</th>
<th>Squared Deviation, ( (x - \bar{x})^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>-5</td>
<td>25</td>
</tr>
<tr>
<td>24</td>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>26</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>26</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>28</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>31</td>
<td>5</td>
<td>25</td>
</tr>
</tbody>
</table>

mean of the squared deviations \( \frac{25 + 4 + 25 + 4}{6} = \frac{58}{6} \approx 9.67 \)

\( \sqrt{9.67} \approx 3.1 \) The standard deviation is approximately **3.1**.

11. Find the standard deviation of 68, 71, 75, 74, 71, 73, 71, 72, 74, and 79.

\[
\text{mean} = \frac{68 + 3(71) + 72 + 73 + 2(74) + 2(75) + 79}{11} = 73
\]

mean of the squared deviations \( \frac{25 + 5(4) + 3(1) + 36}{11} \approx 7.64 \)

\( \sqrt{7.64} \approx 2.8 \) The standard deviation is approximately **2.8**.
12. In Your Turn 11, what is the mean of the deviations before squaring? Use your answer to explain why squaring the deviations is helpful. The mean of the deviations is 0. Squaring the deviations is helpful because it prevents the positive and negative deviations from averaging to 0.

13. How can you determine the first and third quartiles of a data set? First, determine the median. This allows you to determine the lower and upper half of the data set. The first quartile is the median of the lower half of the data set, and the third quartile is the median of the upper half of the data set. Find the median of each half of the data set the same way you find the median of the entire data set.

14. How can you determine the standard deviation of a data set? Find the mean of the data. Then find the square of the distance between each data value and the mean. Find the mean of the squared values. The standard deviation is the square root of this mean.

15. Essential Question Check-In What does the measure of center of a data set indicate? The measure of center identifies either a central or typical value of the data.

Evaluate: Homework and Practice

1. The data set \{13, 24, 14, 15, 14\} gives the times of Tara's one-way ride to school (in minutes) for one week. Is the average (mean) of the times a good description of Tara's ride time? Explain.
   No. The day the trip took 24 minutes will skew the value of the average to be too high.

   Find the mean and median of each data set.

   2. The numbers of hours Cheri works each day are 3, 7, 4, 6, and 5.
   \[\text{mean} = \frac{3 + 7 + 4 + 6 + 5}{5} = \frac{25}{5} = 5\]
   \[\text{median: 3, 4, 5, 6, 7} \quad \text{The median is 5.}\]

   3. The weights in pounds of 6 members of a basketball team are 125, 136, 150, 119, 150, and 143.
   \[\text{mean} = \frac{125 + 136 + 150 + 119 + 150 + 143}{6} = \frac{823}{6} \approx 137.2\]
   \[\text{median: 119, 125, 136, 143, 150, 150} \quad \text{The median is} \frac{136 + 143}{2} = 139.5.\]

   4. 36, 18, 12, 10, 9
   \[\text{mean} = \frac{36 + 18 + 12 + 10 + 9}{5} = \frac{85}{5} = 17\]
   \[\text{median: 9, 10, 12, 18, 36} \quad \text{The median is 12.}\]
5. The average yearly gold price for the period from 2000–2009:
$279.11, $271.04, $309.73, $363.38, $409.72, $444.74, $603.46, $695.39, $871.96, $972.35

\[
\text{mean} = \frac{279.11 + 271.04 + 309.73 + 363.38 + 409.72 + 444.74 + 603.46 + 695.39 + 871.96 + 972.35}{10}
\]
\[= \frac{5220.88}{10} = 522.088, \text{ or } $522.01\]

The median is 409.72 + 444.74 \[\frac{2}{2} = \frac{854.46}{2} = $427.23.\]

6. There are 28, 30, 29, 26, 31, and 30 students in a school’s six Algebra 1 classes.

\[
\text{mean} = \frac{28 + 30 + 29 + 26 + 31 + 30}{6} = \frac{174}{6} = 29\]

The median is 26, 28, 29, 30, 30, 31 \[\text{The median is } \frac{29 + 30}{2} = 29.5.\]

7. The numbers of members in five karate classes are 13, 12, 10, 16, and 19.

\[
\text{mean} = \frac{13 + 12 + 10 + 16 + 19}{5} = \frac{70}{5} = 14\]

The median is 10, 12, 13, 16, 19 \[\text{The median is } 13.\]

8. Find the range and interquartile range for 3, 7, 4, 6, and 5.

\[3, 4, 5, 6, 7\]
Range = 7 − 3 = 4
The interquartile range is IQR = 6.5 − 3.5 = 3.

9. Find the range and interquartile range for 125, 136, 150, 119, and 143.

\[119, 125, 136, 143, 150\]
Range = 150 − 119 = 31
The interquartile range is IQR = 150 − 125 = 25.

10. Find the range and interquartile range for 9, 10, 12, 18, and 36.

\[9, 10, 12, 18, 36\]
Range = 36 − 9 = 27
The interquartile range is IQR = 27 − 9.5 = 17.5.

11. Find the range and interquartile range for 271.04, 279.11, 309.73, 363.38, 409.72, 444.74, 603.46, 695.39, 871.96, and 972.35.

\[271.04, 279.11, 309.73, 363.38, 409.72, 444.74, 603.46, 695.39, 871.96, 972.35\]
\[\text{Range} = 972.35 − 271.04 = 701.31\]
\[\text{The interquartile range is IQR = } 695.39 − 309.73 = 385.66.\]

12. Find the range and interquartile range for 28, 30, 26, 29, 30, 28, 29, 30, 28, 31.

\[26, 28, 29, 30, 30, 31\]
Range = 31 − 26 = 5
The interquartile range is IQR = 30 − 28 = 2.
14. Find the range and interquartile range for 13, 14, 18, 13, 12, 17, 15, and 12.
   \[12, 12, 13, 14, 15, 17, 18\]  
   Range = 18 − 12 = 6
   The interquartile range is IQR = 16 − 12.5 = 3.5.

15. Find the range and interquartile range for 13, 12, 15, 17, and 9.
   \[9, 12, 13, 15, 17\]  
   Range = 17 − 9 = 8
   The interquartile range is IQR = 16 − 10.5 = 5.5.

16. Find the standard deviation of 3, 7, 4, 6, and 5.
   The mean is 5.
   The mean of the squared deviations is \(\frac{4 + 1 + 0 + 1 + 4}{5} = 2\).
   The standard deviation is \(\sqrt{2} \approx 1.4\).

17. Find the standard deviation of 125, 136, 150, 119, 150, and 143.
   The mean is 137.17.
   The mean of the squared deviations is \(\frac{330.03 + 148.03 + 1.36 + 34.02 + 164.69 + 164.69}{6} \approx \frac{842.83}{6} \approx 140.5\).
   The standard deviation is \(\sqrt{140.5} \approx 11.9\).

18. Find the standard deviation of 36, 18, 12, 10, and 9.
   The mean is 17.
   The mean of the squared deviations is \(\frac{64 + 49 + 25 + 1 + 361}{5} = 100\).
   The standard deviation is \(\sqrt{100} = 10\).

19. Find the standard deviation of $279.11, $271.04, $309.73, $363.38, $409.72, $444.74, $603.46, $695.39, $871.96, and $972.35. Round the mean to the nearest $0.01 and the squared deviations to the nearest whole number.
   The mean is 522.09.
   The mean of the squared deviations is \(\frac{63,025 + 59,038 + 45,096 + 25,188 + 12,627 + 5983 + 6621 + 30,034 + 122,410 + 202,736}{10} = 57,258\).
   The standard deviation is \(\sqrt{57,258} \approx 239.3\).

   The mean is 29.
   The mean of the squared deviations is \(\frac{9 + 3(1) + 4}{6} = 2.67\).
   The standard deviation is \(\sqrt{2.67} \approx 1.6\).

21. Find the standard deviation of 13, 14, 18, 13, 12, 17, 15, and 12.
   The mean is 14.25.
   The mean of the squared deviations (rounded).
   \[
   \text{mean} = \frac{2(5.06) + 2(1.56) + 0.06 + 0.56 + 7.56 + 14.06}{8} = 4.44
   \]
   \(\sqrt{4.44} = 2.1\)
   The standard deviation is approximately 2.1.

CRITICAL THINKING
Challenge students to give an example of a data set for which the mean is twice the median. Have students present and explain their answers, then compare the different data sets that students created.
22. Determine whether or not the third quartile has the same value as a member of the data set. Select the correct answer for each lettered part.

A. \{79, 91, 90, 99, 91, 80, 80, 90\}  X Yes \ X No
B. \{98, 96, 96, 91, 81, 87\}  X Yes \ X No
C. \{88, 95, 89, 88, 93, 84, 93, 85, 92\}  X Yes \ X No
D. \{97, 84, 96, 82, 93, 88, 91, 94\}  No \ X Yes \ X No
E. \{94, 85, 97\}  No \ X Yes \ X No
F. \{85, 89, 81, 89, 85, 84\}  No \ X Yes \ X No

Use this data for Exercises 23 and 24. The numbers of members in 6 yoga clubs are 80, 74, 77, 71, 75, and 91.

23. Find the standard deviation of the numbers of members to the nearest tenth.

The mean:
\[
\frac{80 + 74 + 77 + 71 + 75 + 91}{6} = \frac{468}{6} = 78
\]

The mean of the squared deviations is
\[
\frac{49 + 16 + 9 + 1 + 4 + 169}{6} = 41.33.
\]

The standard deviation is \(\sqrt{41.33} \approx 6.4\).

H.O.T. Focus on Higher Order Thinking

24. Explain the Error Suppose a person in the club with 91 members transfers to the club with 71 members. A student claims that the measures of center and the measures of spread will all change. Correct the student’s error.

There are 6 elements in the set, so the IQR is the difference between the 5th element and the 2nd. The only values that are changing are the first and last. This will not affect the IQR. The median is the average of the 3rd and 4th elements and likewise remains unchanged. The mean will also remain unchanged because the sum of all the elements doesn’t change. The range will change from 20 to 18. The standard deviation will also decrease because the value of both the first and last elements will be closer to the mean.

25. What If? If all the values in a set are increased by 10, does the range also increase by 10? Explain.

No, the highest and lowest values both increase by 10, so their difference, the range, will stay the same.

26. Communicate Mathematical Ideas Jorge has a data set with the following values: 92, 80, 88, 95, and \(x\). If the median value for this set is 88, what must be true about \(x\)? Explain.

\(x \leq 88\); for 5 ordered values, the median is the 3rd. Since 95 and 92 \(\geq 88\), they must be 4th and 5th, so \(x\) must be \(\leq\) the median.

27. Critical Thinking If the value for the median of a set is not found in the data set, what must be true about the data set? Explain.

The data set must have an even number of values. When a data set has an odd number of values, the median is always found in the data set.
Lesson Performance Task

The table lists the ages of the soprano and bass singers in a town choir. Find the mean, median, range, interquartile range, and standard deviation for each type of singer in the data set. Interpret each result. What can you conclude about the ages of the different types of singers?

<table>
<thead>
<tr>
<th>Age of Soprano Singers</th>
<th>63</th>
<th>42</th>
<th>28</th>
<th>45</th>
<th>36</th>
<th>48</th>
<th>32</th>
<th>40</th>
<th>57</th>
<th>49</th>
</tr>
</thead>
</table>

| Age of Bass Singers | 32 | 34 | 53 | 35 | 43 | 41 | 29 | 35 | 24 | 34 |

Soprano Singers:
Mean age of sopranos: \( \frac{63 + 42 + 28 + 45 + 36 + 48 + 32 + 40 + 57 + 49}{10} = \frac{440}{10} = 44 \)
Median age of sopranos: \( \frac{42 + 45}{2} = \frac{87}{2} = 43.5 \)
Range of ages of sopranos: \( 63 - 28 = 35 \)
Interquartile range of ages of sopranos: \( IQR = 49 - 36 = 13 \)
Standard deviation:
\[
\sqrt{\frac{61 + 4 + 256 + 1 + 64 + 16 + 44 + 16 + 169 + 25}{10}} = \sqrt{\frac{1056}{10}} = \sqrt{105.6} \approx 10.3
\]

Bass Singers:
Mean bass singer age: \( \frac{32 + 34 + 53 + 35 + 43 + 41 + 29 + 35 + 24 + 34}{10} = \frac{360}{10} = 36 \)
Median bass singer age: \( \frac{34 + 35}{2} = \frac{69}{2} = 34.5 \)
Range of bass singer ages: \( 53 - 24 = 29 \)
Interquartile range of bass singer ages: \( IQR = 41 - 32 = 9 \)
Standard deviation:
\[
\sqrt{\frac{16 + 4 + 289 + 1 + 49 + 25 + 49 + 1 + 144 + 4}{10}} = \sqrt{\frac{582}{10}} = \sqrt{58.2} = 7.6
\]

The age of a soprano singer is generally greater than the age of a bass singer.
The ages of the bass singers are more tightly clustered than the ages of the soprano singers.

EXTENSION ACTIVITY

Have students consider the following scenario. A 17-year-old high school student and his 55-year-old father both join the bass section of the choir. How would adding their ages to the data set given in the Lesson Performance Task affect your conclusions?

Students should find that the mean, median, and interquartile range of the ages in the bass section would remain the same, but the range would increase to 38, and the standard deviation would increase to 10.4. The soprano section would still have the greater typical age, but the ages of the bass singers would be more spread out than the ages of the sopranos.

CONNECT VOCABULARY

Some students may not be familiar with the terms soprano, alto, tenor, and bass. Explain that they describe types of singing voices, with the soprano section of a choir singing the highest notes and the bass section singing the lowest. Ask for volunteers in the class who sing in a choir to explain or demonstrate these four vocal ranges.

QUESTIONING STRATEGIES

Which of the statistics that you calculated are measures of center? What does comparing each of these measures for the sopranos to the same measures for the basses tell you about the two groups? Mean and median; comparing tells you whether the typical age of a soprano is greater or less than the typical age of a bass.

Which of the statistics that you calculated are measures of spread? What does comparing each of these measures for the sopranos to the same measures for the basses tell you about the two groups? Range, interquartile range, and standard deviation; it tells you whether the ages of the sopranos are closer together or farther apart than the ages of the basses.

Scoring Rubric

2 points: Student correctly solves the problem and explains his/her reasoning.
1 point: Student shows good understanding of the problem but does not fully solve or explain his/her reasoning.
0 points: Student does not demonstrate understanding of the problem.